Temperature prediction in domestic refrigerator: Determinist and stochastic approaches

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TEMPERATURE PREDICTION IN DOMESTIC REFRIGERATOR: DETERMINIST AND STOCHASTIC APPROACHES

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ABSTRACT

The influence of two random variables: ambient temperature and thermostat setting, on air and load temperatures in non ventilated domestic refrigerator was studied. A simplified steady state heat transfer model was developed which takes into account heat exchanges by convection and radiation. This model considers circular airflow due to natural convection inside the cavity. The normal distribution of ambient and thermostat temperatures was developed to fit the survey data. Monte Carlo method was used for a sampling of ambient and thermostat temperatures from this distribution. These values were then used as input parameters of the deterministic model allowing the calculation of air and load temperatures. An analysis of the predicted temperatures was undertaken by comparing with survey data of air and load temperatures in domestic refrigerators in France, a good agreement was obtained. This model can be used as risk evaluation tool.

1. INTRODUCTION

An audit of cold chain in France (Cemagref and ANIA, 2004) was carried out on product temperature monitoring along the cold chain from production to consumption. It was found that 40% of product is not preserved at an appropriate temperature in domestic refrigerator. Most of temperature prediction in domestic refrigerator is carried out by deterministic models which assume that the coefficients, initial and operating conditions are accurately known. However, due to the consumer practice (thermostat setting, ambient temperature, loading etc), the operating condition is subjected to random variation. As a consequence, the air and load temperatures in refrigerator are also random and should be considered as stochastic variables which are characterised by statistical values such as mean, variance and probability density.
Several numerical studies have been carried out on heat transfer in empty domestic refrigerators (Pereira and Nieckele, 1997; Silva and Melo, 1998; Deschamps et al, 1999). However, few studies have been carried out on loaded refrigerators. The numerical studies provide knowledge on the temperature and velocity heterogeneity under determined conditions. Laguerre et al (2007) carried out CFD simulation (Fluent software) on empty and loaded static refrigerator (without a fan) taking into account radiation. In this study, circular airflow along the refrigerator walls and temperature stratification along the height are shown. In spite that CFD is a powerful simulation tool; its use is limited because of calculation time (several days for one calculation using more than 1 million of cells for a loaded refrigerator).
The coupling of deterministic and stochastic models is largely used in food process engineering due to the variation of product biological properties and uncertain process conditions. This approach is applied, for example, heat treatment of packaged foods (Nicolaï et al. 2000, Baucour et al, 2003), Different methods have been proposed in these studies to quantify the effects of the uncertainty of model parameters on the output of the studied system. In cold chain, product is often exposed to uncertain environment such as temperature and duration in refrigerating equipment. Bogataj et al (2005) studied the effect of time-temperature evolution in the cold chain on product deterioration. Dabbene et al (2008a,b) proposed an approach for optimisation of fresh food quality (ripeness, microbial charge, product temperature) and logistic cost.
The objective of this work was to study the influence of two random variables: ambient temperature and thermostat setting, on air and load temperature in domestic refrigerator. A simplified heat transfer model was developed and a comparison of simulation results with the ones obtained by surveys was carried out.

2. SIMPLIFIED HEAT TRANSFER MODEL

A proposed simplified heat transfer model of loaded refrigerator represents the main phenomena observed by CFD simulation: circular airflow in the cavity, temperature stratification along the height. Steady state is assumed in refrigerator. Figure 1 presents a diagram of simplified airflow and heat transfer. During the flow, air exchanges heat with cold wall, with bottom load, with warm wall and with top load, simultaneously. There is also radiation between cold wall and load and between warm wall and load. Finally, there is conduction in the door and the side walls and convection with the environment. The model developed in this study allows the calculation of air temperature and load temperatures in function of the room temperature (T_e) and the thermostat temperature (T_{th}).

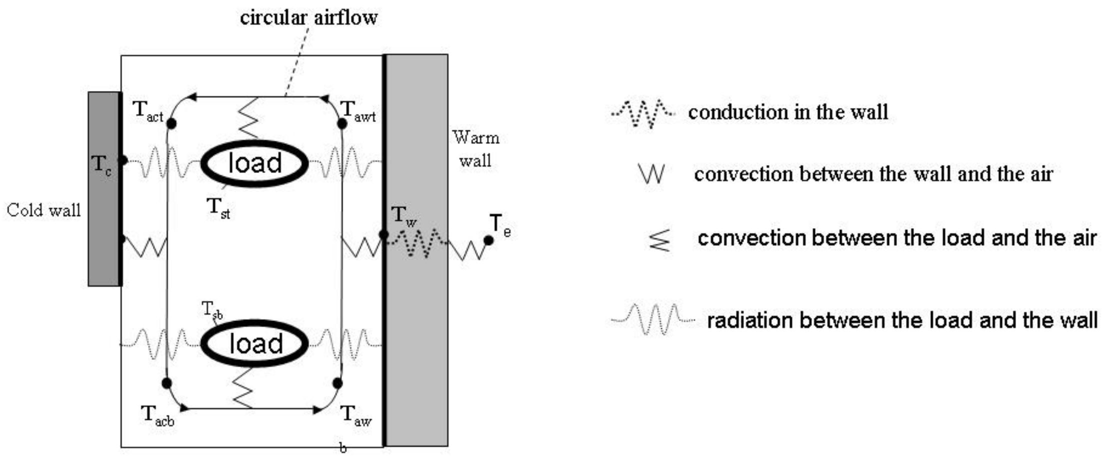


Figure 1: Simplified model of heat transfer and airflow in loaded refrigerator.

2.1. Equation development

Heat exchanges between air and cold/warm walls.

Near the cold wall, air (inlet temperature T_{act}) exchanges heat with this wall (at constant temperature T_c) with a convective heat transfer coefficient h_c. This exchange leads to cool down the air (outlet air temperature T_{acb}).

$$\dot{m}Cp\, dT_a = h_c(T_c - T_a)\, dA \Rightarrow \ln\left(\frac{T_{acb} - T_c}{T_{act} - T_c}\right) = \left(-\frac{h_c A_c}{\dot{m}C_p}\right) \Leftrightarrow$$

$$(T_{acb} - T_c) = \alpha_c (T_{act} - T_c) \tag{1}$$

The same phenomena take places near the warm wall.

$$(T_{awt} - T_w) = \alpha_w (T_{awb} - T_w) \tag{2}$$

Heat exchanges between air and load.

The air (inlet temperature T_{act}) exchanges heat with the top load (temperature T_{st}) with a convective heat transfer coefficient h_s. This exchange leads to the exit air temperature T_{act}.

$$(T_{act} - T_{st}) = \alpha_s (T_{awt} - T_{st}) \tag{3}$$

The same phenomena take place between air and bottom load.

$$(T_{awb} - T_{sb}) = \alpha_s (T_{acb} - T_{sb}) \tag{4}$$

Heat balance on warm wall.
There is convective heat transfer between warm wall and air: inlet air temperature T_{awb} and outlet air temperature T_{awt}. There is radiative heat transfer between warm wall and the top and bottom loads. The radiative heat flux is proportional to $T_{st}^4 - T_w^4$ for the top load and $T_{sb}^4 - T_w^4$ for the bottom load. But since the temperature differences (T_{st}-T_w) and (T_{sb}-T_w) are small compared to absolute temperature (T_{sh}, T_{sb}, T_w), the radiative heat transfer can be considered as almost proportional to the temperature differences.
In steady state, the convective and radiative heat fluxes on the warm wall is balanced with heat flux from this wall to ambience with the coefficient h_e.

$$\dot{m}C_p(T_{awb} - T_{awt}) + h_{rwt}A_{rwt}(T_{st} - T_w) + h_{rwb}A_{rwb}(T_{sb} - T_w) = h_e A_e (T_w - T_e) \quad \Leftrightarrow$$

$$(T_{awb} - T_{awt}) + \beta_{rwt}(T_{st} - T_w) + \beta_{rwb}(T_{sb} - T_w) = \beta_e (T_w - T_e) \tag{5}$$

Heat balance on the top load.
There is convective heat exchange between the top load and air and radiative heat exchange between the load and the cold and warm walls.

$$\dot{m}C_p(T_{awt} - T_{act}) + h_{rct}A_{rct}(T_c - T_{st}) + h_{rwt}A_{rwt}(T_w - T_{st}) = 0 \quad \Leftrightarrow$$

$$(T_{awt} - T_{act}) + \beta_{rct}(T_c - T_{st}) + \beta_{rwt}(T_w - T_{st}) = 0 \tag{6}$$

Heat balance on the bottom load:
The same phenomena take place on the bottom load.

$$(T_{acb} - T_{awb}) + \beta_{rcb}(T_c - T_{sb}) + \beta_{rwb}(T_w - T_{sb}) = 0 \tag{7}$$

In our study, it is supposed that the room temperature (T_e) and air temperature near the thermostat (T_{th}) are known. Since, the thermostat sensor is generally located at the bottom of the back wall; it is reasonably to represent T_{th} by T_{acb}. The eq. 1 to 7 can be expressed as:

$$\mathbf{A}.T = \mathbf{B}.T_e + \mathbf{C}.T_{th} \tag{8}$$

Where **A, B** and **C** are matrix of dimensionless coefficients and **T** is matrix of temperature to be predicted.

2.2. Coefficient estimation

Estimation of mass flow rate of air
To our knowledge, there is no correlation in literature allowing the estimation of air flow rate ($\dot{m}$) in a loaded refrigerator as a function of different wall temperatures. Therefore it is proposed to estimate $\dot{m}$ in a simple well known situation: that of an empty cavity with the same dimension of cold and warm walls. These walls are differentially heated. Considering a natural convection heat transfer coefficient of about 5 $W.m^{-2}.K^{-1}$ (order of magnitude of heat transfer coefficient by natural convection, Ben Amara, 2005) with a typical wall surface of 0.5 m^2, the estimated air flow rate $\dot{m}$ is about 0.0025 $kg.s^{-1}$ (Raithby and Hollands, 1992).

Estimation of heat transfer coefficients
The estimation of coefficients h_c, h_w, h_r and h_e was presented in our previous study (Laguerre and Flick, 2004). The numerical values used in estimation of the heat transfer coefficients are shown in Table 1.

Table 1: Numerical values used in estimation of dimensionless heat transfer coefficients.

h_c	3.28 $W.m^{-2}.K^{-1}$	
h_w	1.3 $W.m^{-2}.K^{-1}$	

h_r	3.85 $W.m^{-2}.K^{-1}$	Laguerre and Flick (2004)
h_e	0.63 $W.m^{-2}.K^{-1}$	
h_s	3.26 $W.m^{-2}.K^{-1}$	Raithby and Hollands (1992)
Cp	1005 $J.kg^{-1}.K^{-1}$	Loncin (1985)
$\dot{m}$	$2.5x10^{-3}$	Raithby and Hollands (1992)
A_c	0.15 m^2	
A_w	1.65 m^2	
A_s	0.50 m^2	

2.3. Numerical resolution

The numerical values used in the estimation of the heat transfer coefficients are shown in Table 1. Finally, it is possible to calculate A, B and C so that eq. 8 can be numerically expressed as follows:

$$
\begin{aligned}
(T_c - T_{th}) &= -0.368(T_e - T_{th}) \\
(T_{act} - T_{th}) &= 0.080(T_e - T_{th}) \\
(T_{awt} - T_{th}) &= 0.086(T_e - T_{th}) \\
(T_{awb} - T_{th}) &= 0.004(T_e - T_{th}) \\
(T_w - T_{th}) &= 0.147(T_e - T_{th}) \\
(T_{sb} - T_{th}) &= 0.008(T_e - T_{th}) \\
(T_{st} - T_{th}) &= 0.072(T_e - T_{th})
\end{aligned}
$$

3. PROPOSED METHODOLOGY COMBINING DETERMINISTIC AND STOCHASTIC MODELLING

The proposed methodology for the prediction of air and load temperature distributions is shown in Figure 2. Values of T_{th} and T_e are randomly withdrawn from their respective distributions obtained from survey data (Laguerre et al, 2002; Hunt and Gidman, 1982). The Monte Carlo method is used for this random withdraw. T_{th} and T_e values are used as input random parameters in the thermal model, and the values of T_c, T_w, T_{act}, T_{awt}, T_{awb}, T_{st}, T_{sb} are calculated. These two steps are repeated for example 1000 times. In all cases, a comparison between calculated air and product temperatures with survey result (Laguerre et al, 2002; Cemagref and ANIA, 2004) can finally be undertaken.

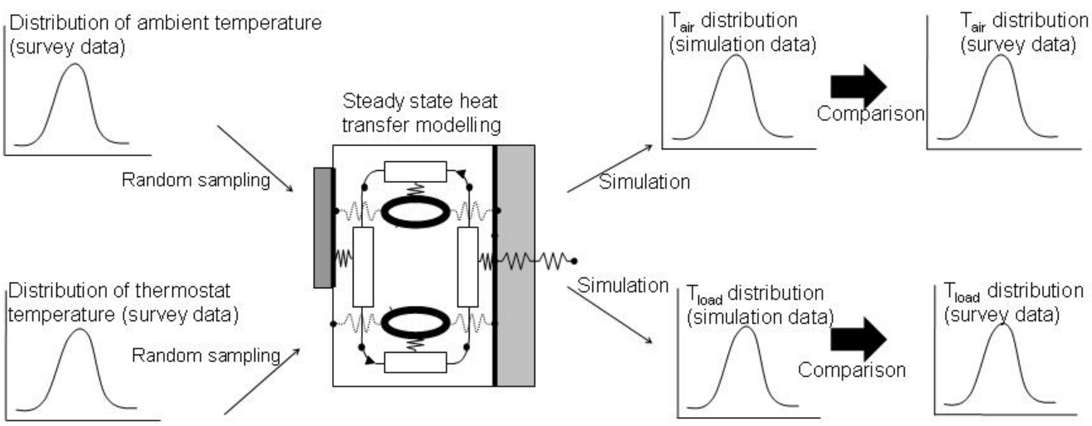
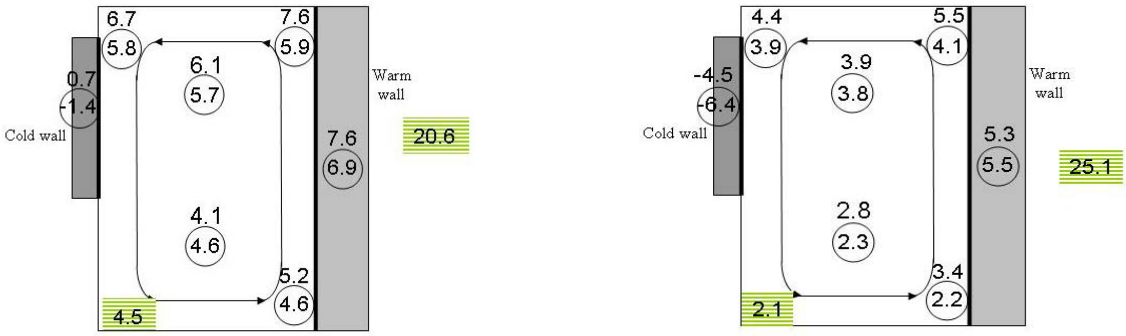


Figure 2: Proposed methodology combining deterministic and stochastic modelling.

4. RESULTS AND DISCUSSION

4.1. Model validation

Experiments were undertaken on a refrigerator without a fan loaded with packages of test product made of methylcellulose. These packages were evenly spread on the glass shelves; some of them were instrumented in the core using thermocouples (T-type). Two experimental conditions were used: the first involved a room temperature of 20.6°C and a medium thermostat setting (air temperature measured near the thermostat: 4.5°C), the second involved a room temperature of 25.1°C and a low thermostat setting (temperature of 2.1°C). The air, product and wall temperatures were recorded over a period of 4 h after steady state condition had been reached (≥24 h of operation). Then the time-averaged values were calculated for each measurement point. For comparison with the predicted value, the mean temperature of the top and the bottom loads was estimated by taking the mean value of 6 instrumented packages (Figure 3). Using the above equations lead to the calculated temperatures. In the case of T_{th} of 4.5°C and T_e of 20.6°C (Figure 3a), the calculated air temperature varies from 4.5°C at the bottom to 5.9°C at the top while the load temperature varies from 4.6°C at the bottom to 5.7°C at the top. The results for a thermostat temperature of 2.1°C and a room temperature of 25.1°C are shown in Figure 3b. It can be seen that the model most often underestimates the experimental temperature: maximum difference 0.5°C for the load temperature and 1.7°C for the air temperature. This can be explained by the model simplification and the uncertainties of the estimated coefficients.



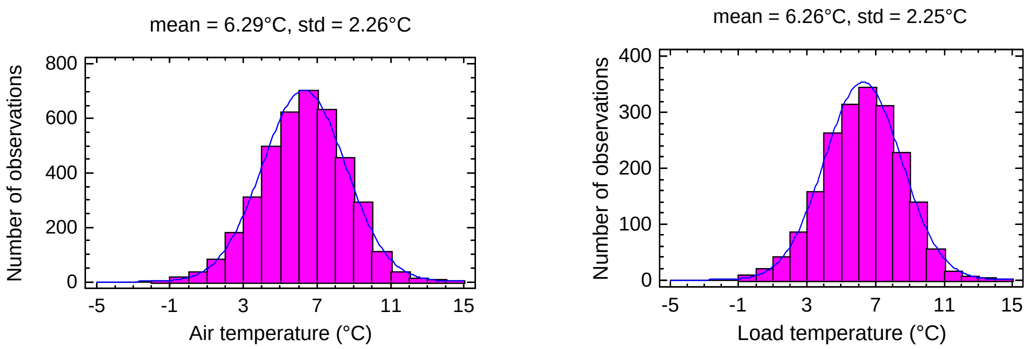
(a) (b)

Figure 3: Comparison between the experimental (value above the circle) and the calculated temperatures (value in the circle) for the 2 input parameters: (a) thermostat temperature of 4.5°C and room temperature of 20.6°C (b) thermostat temperature of 2.1°C and room temperature of 25.1°C.

4.2. Comparison between survey temperatures and the ones predicted by deterministic and stochastic approaches

The air temperature is predicted for 1000 simulations according to the methodology cited in 4. The distribution curve (Figure 4a) shows a mean value of 6.3°C (standard deviation 2.3°C) while the survey result shows a mean value of 6.6°C (standard deviation 2.6°C). This difference can be explained by the lower room temperature used in the simulation. Indeed, the room temperature distribution law was established from the survey carried out in the UK in winter (February and March).

Figure 4b presents load temperature obtained from 1000 simulations. The distribution curve shows a mean value of 6.3°C (standard deviation 2.3°C) while the survey result shows a mean value of 5.9°C (standard deviation 3.0°C). This slight difference could be explained by the choice of the simplified heat transfer model and its parameters proposed in this study. It could also be explained by the fact that some load temperatures in the survey were negative (products perhaps stored in the ice compartment instead of the refrigerator compartment of the refrigerator). It is to be emphasized that the calculated load temperature may vary from 1.7°C to 10.9°C (mean ± 2 x standard deviation) with a 95% confidence interval. It is also noteworthy that for sensitive products such as meat and meat products, the recommended preservation temperature is 0 to 4°C. The predicted results show that the load temperature is above 4°C for 76% of the observations.



(a) (b)

Normal distribution with the same mean and standard deviation

Figure 4: Distribution of air (a) and load (b) temperatures in refrigerator obtained from 1000 simulations. Surveyed results: mean air temperature 6.6°C (standard deviation 2.6°C) and mean load temperature 5.9°C (standard deviation 3.0°C).

5. CONCLUSION

This study proposed combined deterministic and stochastic approaches to predict the air and load temperatures in static domestic refrigerators (without a fan). In such refrigerators, natural convection and radiation are important heat transfer modes. A simplified steady state heat transfer model was developed and two random variables were used as input parameters: room and thermostat temperatures. In fact, the room temperature varies due to seasons and the thermostat setting varies according to consumer behaviour. These two temperatures were deduced using a distribution law constructed from the survey results.
This model considers ~~a~~ circular airflow in the cavity and heat transfer by convection and radiation between the air, the walls and the load. To take into account temperature stratification, this model allows calculation of temperatures at the top and bottom of the cavity.
A comparison between the predicted and survey air and load temperatures shows a good agreement. The slight difference can be explained, on one hand, by underestimation of the room temperature (survey carried out during winter) and on the other hand, by the difficulty encountered when attempting to accurately estimate the heat transfer coefficients.
It is to be reminded that the model was developed based on a given refrigerator design (single door, without a fan). In future work, this approach will be applied to several refrigerator designs. In a longer term study, the methodology developed will be extended to other refrigerating equipment such as cold rooms, display cabinets and refrigerated trucks. Knowledge of the load time-temperature history can be used in combination with a predictive microbiological model to evaluate the contamination level throughout the cold chain. In this manner, the present study paves the way to a numerical risk evaluation tool. The model could be used to evaluate the influence of environmental temperatures and operating conditions on the product temperature evolution along the cold chain (sensitivity study). As an example, the model could predict the influence of room temperature in tropical countries

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NOMENCLATURE

A_c	Surface of evaporator (cold wall) participating to convective heat transfer (m^2)
A_w	Surface of warm wall participating to convective heat transfer (m^2)
A_r	Wall surface participating to heat exchange by radiation (m^2)
A_e	External surface participating to convective heat transfer with ambience (m^2)

C_p	Air heat capacity ($J.kg^{-1}.K^{-1}$)
g	Acceleration due to gravity = 9.81 ($m.s^{-2}$)
h	Heat transfer coefficient ($W.m^{-2}.K^{-1}$)
$\dot{m}$	Mass flow rate of air in refrigerator ($kg.s^{-1}$)
T_{ab}	Air temperature at the bottom of cavity (K)
T_{at}	Air temperature at the top of cavity (K)
T_{acb}	Air temperature at the bottom near the cold wall (K)
T_{act}	Air temperature at the top near the cold wall (K)
T_{awb}	Air temperature at the bottom near the warm wall (K)
T_{awt}	Air temperature at the top near the warm wall (K)
T_c	Cold wall temperature (K)
T_e	Room temperature (K)
T_{st}	Load temperature at the top of cavity (K)
T_{sb}	Load temperature at the bottom of cavity (K)
T_{th}	Air temperature near thermostat sensor (K)
T_w	Warm wall temperature (K)

Greek symbol

λ	Thermal conductivity ($W.m^{-1}.K^{-1}$)
β	Thermal expansion coefficient (K^{-1})
ρ	Density ($kg.m^{-3}$)
ν	Kinematic viscosity ($m^2.s^{-1}$)
$\alpha_c = e^{-\beta_c}$	Dimensionless convective heat transfer between air and cold wall, $\beta_c = \frac{h_c A_c}{\dot{m} C_p}$
$\alpha_w = e^{-\beta_w}$	Dimensionless convective heat transfer between air and warm wall, $\beta_w = \frac{h_w A_w}{\dot{m} C_p}$
$\alpha_s = e^{-\beta_s}$	Dimensionless convective heat transfer between air and load, $\beta_s = \frac{h_s A_s}{\dot{m} C_p}$
β_e	Dimensionless convective heat transfer between warm wall and ambience $= \frac{h_e A_e}{\dot{m} C_p}$
β_{rct}	Dimensionless radiative heat transfer between cold wall and load located at the top of cavity $= \frac{h_{rct} A_{rct}}{\dot{m} C_p}$

β_{rcb}	Dimensionless radiative heat transfer between cold wall and load located at the bottom of cavity $= \frac{h_{rcb} A_{rcb}}{\dot{m} C_p}$
β_{rwt}	Dimensionless radiative heat transfer between warm wall and load located at the top of cavity $= \frac{h_{rwt} A_{rwt}}{\dot{m} C_p}$
β_{rwb}	Dimensionless radiative heat transfer between warm wall and load located at the bottom of cavity $= \frac{h_{rwb} A_{rwb}}{\dot{m} C_p}$

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